Solution : DTS

1.(C) $\vec{B} = 60 \sin(\pi \times 10^3 x + 3\pi \times 10^{11} t)\, \hat{k}$

Comparing with $\vec{B} = 60 \sin(kx + \omega t)\, \hat{k}$, we see that $k = \pi \times 10^3 \ m^{-1}$

Wavelength, $\lambda = \frac{2\pi}{k} = 2 \times 10^{-3} \ m = 2 \ mm$

2.(A) Polarisation, using polariser and analyser, we get change in intensity of light at different rotations, hence we conclude that light waves are transverse in nature.

3.(B) $c = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$ and $c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\quad \therefore \quad$ Refractive index $= \frac{c_0}{c} = \sqrt{\mu_r \varepsilon_r}$

4.(A) The dielectric constant is another term for the relative permittivity of the medium, i.e. ε_r.

We know that the refractive index of a medium,

$$n = \sqrt{\varepsilon_r \mu_r} \quad \Rightarrow \quad \mu_r = \frac{n^2}{\varepsilon_r} = \frac{(1.5)^2}{1.8} = 1.25$$

5.(D) Amplitude of the magnetic field, $B_0 = \frac{E_0}{c} = \frac{E_0}{\left(\frac{\omega}{k}\right)} = E_0\left(\frac{k}{\omega}\right)$

Direction of propagation, $\hat{S} = \hat{k}$

Therefore, direction of oscillation of magnetic field,

$$\hat{B} = \hat{S} \times \hat{E} = \hat{k} \times \hat{i} = \hat{j}$$

Also, the electric field and the magnetic field of an EM wave are always perfectly in phase.

6.(C) Direction of propagation, $\hat{S} = \hat{i}$

Therefore, direction of oscillation of electric field,

$$\hat{E} = \hat{B} \times \hat{S} = \left(-\hat{j}\right) \times \hat{i} = \hat{k}$$

7.(B) $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^9} = 3 \times 10^{-1} = 0.3 \ m = 30 \ cm$$

8.(B) Intensity of an EM wave, $I = \frac{B_{rms}^2 c}{\mu_0 \mu_r} = \left(\frac{1}{\mu_0 \mu_r}\right)\left(\frac{c_0}{\sqrt{\varepsilon_r \mu_r}}\right) B_{rms}^2 = \left(\frac{3 \times 10^8}{\left(\sqrt{1.44}\right)\left(4\pi \times 10^{-7}\right)}\right)\left(10^{-6}\right)^2 = \frac{625}{\pi} \ W / m^2$

9.(A) $I = \varepsilon_0 E_{rms}^2 c = \frac{\varepsilon_0 E_{rms}^2}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} E_{rms}^2$

$I = \frac{B_{rms}^2 c}{\mu_0} = \frac{B_{rms}^2}{\mu_0 \sqrt{\varepsilon_0 \mu_0}}$

10.(B) Comparing the given equation with the equation of plane electromagnetic wave,

$$E_z = E_0 \cos(\omega t + kx)$$

We have $\quad \omega = 6 \times 10^8$ and $k = 4$

Velocity of light in medium,

$$v = \frac{\omega}{k} = \frac{6\times10^8}{4} = \frac{3}{2}\times10^8 m/s$$

$\therefore$ Refractive Index, $\mu = \frac{c}{v} = \frac{3\times10^8}{\frac{3}{2}\times10^8} = 2$

11.(D) The direction of propagation must be along the unit vector, $\hat{S} = \hat{E}\times\hat{B}$

Also the unit vectors $\hat{E}$ *and* $\hat{B}$ must be perpendicular to each other.

In this question, it is best to draw the given unit vectors and then apply the above two conditions.

12.(B) $E_y = 2.5\frac{N}{C}\left[\left(2\pi\times10^6\frac{rad}{s}\right)t - \left(\pi\times10^{-2}\frac{rad}{m}\right)x\right]$; $E_z = 0$

The wave is moving in the positive direction of *x*.

This is the form $E_y = E_0(\omega t - kx)$

$$\omega = 2\pi\times10^6$$

$2\pi v = 2\pi\times10^6$ or $v = 10^6 Hz$

$$\frac{2\pi}{\lambda} = k$$

or $\frac{2\pi}{\lambda} = \pi\times10^{-2}$ or $\lambda = \frac{2\pi}{\pi\times10^{-2}} = 2\times10^2 = 200m$

13.(C) For a non-magnetic medium, $\mu_r = 1$

Therefore, the refractive index, $n = \sqrt{\varepsilon_r\mu_r} = \sqrt{4(1)} = 2$

Wavelength, $\lambda' = \frac{\lambda}{n} = \frac{\lambda}{2}$

and wave velocity, $v = \frac{c}{n} = \frac{c}{2}$

Hence, it is clear that wavelength and velocity will become half but the frequency remains unchanged.

14.(B) Initial momentum of surface, $\rho_i = E/c$

where, c = velocity of light (constant)

Since, the surface is perfectly reflecting, so the same momentum will be reflected completely. Find momentum,

$pf = -E/c$ [negative value]

$\therefore$ Change in momentum,

$$\Delta p = p_i - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

15.(A)

Solution : JEE Main (Archive)

1.(C) Refractive index, $\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = 2$

Speed and wavelength of wave will becomes half, the frequency remaining unchanged (frequency of a wave depends on the source as due to refraction, it is assumed that the energy is conserved. hv remains the same)

2.(B) $U_{av} = \varepsilon_0 E_{rms}^2 = 4.58 \times 10^{-6}\ J/m^3 \Rightarrow U_{avg} = 4.58 \times 10^{-6}\ J/m^3$

3.(C) Frequency of radio wave is less than that of microwave.

4.(B) FACTUAL

5.(A) Speed of wave $\omega/k = 50/4 \times 10^{-7} = 12.5 \times 10^7 m/s$ $\therefore$ $k = \frac{3 \times 10^8}{12.5 \times 10^7} = \frac{30}{12.5} = 2.4$

6.(D) FACTUAL

7.(A) $E_0 = CB_0$

$= 3 \times 10^8 \times 20 \times 10^{-9} = 6\ V/m$

8.(A) Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 C$, where E_0 is amplitude of the electric field of the light.

$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 C$

$E_0 = \sqrt{\frac{2P}{4\pi r^2 C E_0}} = 2.4\ V/m$

9.(A) FACT

10.(B) Infrared waves $\to$ To treat muscular strain

Radio waves $\to$ for broadcasting

X-rays $\to$ To detect fracture of bones

Ultraviolet rays $\to$ Absorbed by the ozone layer of the atmosphere.

11.(B) $U_{average}/V = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}(8.85 \times 10^{-12})(4)^2 = 35.2 \times 10^{-12} J/m^3$

12.(D) $\frac{V_{air}}{V_{med}} = \frac{c}{c/2} = 2 = \frac{\sqrt{\mu_0 \epsilon_0 \mu_{r_2} \epsilon_{r_2}}}{\sqrt{\mu_0 \epsilon_0 \mu_{r_1} \epsilon_{r_1}}}$; $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

13. (B) $U_F = U_B$ (Matter of fact)

14.(C) Poynting vector, $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$, gives the direction of propagation of em wave

Since $\vec{S}$ is along positive x, $\vec{E}$ along positive y, so $\vec{B}$ is along $+z$-direction.

Using $C = \frac{E}{B} \Rightarrow B = \frac{E}{C} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8} T$ and $\boxed{\vec{B} = (2.1 \times 10^{-8} T)\hat{k}}$ i.e. option (C)

15.(C) $\vec{E} = E_0(\vec{k}.\vec{r} - \omega t)$

$\vec{E}(x, y) = 10\hat{j}\cos(6x + 8z) = 10 j \cos\left[(6\hat{i} + 8\hat{k}) \cdot (x\hat{i} + z\hat{k})\right]$

$\vec{k} = 6\hat{i} + 8\hat{k},\ \omega = |\vec{k}|\, c = 10c$

$\vec{E}(x,y,z,t) = 10\hat{j}\cos\left[\left(6\hat{i}+8\hat{k}\right)\cdot\left(x\hat{i}+z\hat{k}\right)-10ct\right]$

Now, $\vec{E}\times\vec{B} = \dfrac{E^2}{c}\hat{k} = \dfrac{E^2}{10c}\left(6\hat{i}+8\hat{k}\right)$

$$10\hat{j}\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right) = \frac{10}{c}\left(6\hat{i}+8\hat{k}\right)$$

Looking at the options we can say

$$\vec{B} = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)$$

$$\vec{B} = (x,y,z,t) = \frac{1}{c}\left(6\hat{k}-8\hat{i}\right)\cos\left[(6x+8z)-10ct\right]$$

16.(A) $E_{\max} = CB_{\max}$

$$= 3\times10^8\times100\times10^{-6}$$

$$= 3\times10^4$$

17.(D) Since Intensity : $I = \dfrac{P}{A} = \dfrac{1}{2}\in_0 E_0^2 C \quad \Rightarrow \quad$ *Electric field :* $E_0 = \sqrt{\dfrac{2P}{A\in_0 C}}$

$$E_0 = \sqrt{\frac{27\times10^{-3}\times2}{10\times10^{-6}\times9\times10^{-12}\times3\times10^8}}$$

$$E_0 = 1.4\ KV/m.$$

18.(B) Intensity of an EM wave (in terms of E) is given by:

$$I_0 = \frac{1}{2}\in_0 E_0^2 C$$

In medium $I = \dfrac{1}{2}\in_r\in_0 E^2\dfrac{C}{n}$

Also, for EM waves: $n \approx \sqrt{\in_r}$ $\qquad (\because \mu_r \approx 1)$

So, equating I & I_0

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} \Rightarrow \frac{E_0}{E} = \sqrt{n}$$

Similarly: $I_0 = \dfrac{1}{2\mu_0}B_0^2.C$

$$I = \frac{1}{2\mu_r\mu_0}B^2\frac{C}{n}$$

Equating & using $\mu_r \approx 1$

$$\frac{B_0}{B} = \frac{1}{\sqrt{n}}$$

19.(A) $< I > = \in_0 E_{rms}^2.C$

$E_{rms} = C.B_{rms}$

Putting values

B comes in order of $10^{-4}T$

20.(C) Energy Density = $\frac{1}{2}\varepsilon_0 E^2$

Energy in unit area

$= \frac{1}{2}\varepsilon_0 E^2 \times$ Velocity

$I = \frac{1}{2}\varepsilon_0 E^2 \times C$

$I' = \frac{1}{2}\varepsilon E^2 \times V$

$I' = 0.96I \quad \Rightarrow \quad \frac{1}{2}\varepsilon E^2 V = 0.96 \times \frac{1}{2}\varepsilon_0 E_0^2 C \quad \Rightarrow \quad E = \sqrt{0.96} \times \sqrt{\frac{\varepsilon_0}{\varepsilon_r \varepsilon_0} \times \frac{C}{V}} \times E_0$

$= \sqrt{0.96} \times \sqrt{\frac{1}{n^2} \times n}\ E_0$

$= \sqrt{\frac{0.96}{n}} \times E_0 = 24\ V/m$

Here , $n = \sqrt{\mu_r \varepsilon_r}$ and $\mu_r = 1$ for EM waves

21.(C) $\frac{E}{B} = c \quad \Rightarrow \quad B = \frac{6}{3 \times 108} = 2 \times 10^{-8} T$

Direction of magnetic field will be perpendicular to both electric field and direction of propagation of wave

22.(B) $\vec{B} = 1.6 \times 10^{-6} \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(2\hat{i} + \hat{j}\right)\frac{Wb}{m^2}$

$E_0 = cB_0$

$= 3 \times 10^8 \times 1.6 \times 10^{-6} = 4.8 \times 10^2$

From the equation of $\vec{B}$ we can conclude that the direction of wave is $-\hat{k}$

Direction of $\vec{E} =$ direction of $\vec{B} \times \vec{v}$

$= \left(2\hat{i} + \hat{j}\right) \times \left(-\hat{k}\right) = -\hat{i} + 2\hat{j}$

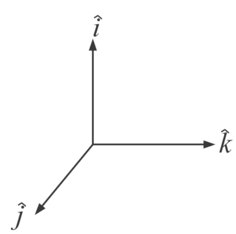
Hence $\vec{E} = 4.8 \times 10^2 \cos\left(2 \times 10^7 z + 6 \times 10^{15} t\right)\left(-\hat{i} + 2\hat{j}\right) V/m$

23.(C) $B_{net} = \sqrt{B_0^2 + B_1^2} = 10^{-6} \times \sqrt{904} \quad \therefore \quad E = C \times B_{net} = 3 \times 10^8 \times 10^{-6} \times \sqrt{904}$

$\therefore \quad F = QE = 10^{-4} \times 3 \times 10^8 \times 10^{-6} \times \sqrt{904} = 0.9\ N. \quad \therefore \quad F_{rms} = \frac{F}{\sqrt{2}} = 0.6N$

24.(A) $\vec{E} = E\hat{i}\cos(kz)\cos(wt)$

It is made by superposition of 2 waves.



$\vec{E}_1 = \frac{E_0}{2}\hat{i}\cos(kz - wt)$

$\vec{E}_2 = \frac{E_0}{2}\hat{i}\cos(kz + wt)$

Corresponding $\vec{B}$

$\vec{B}_1 = \frac{E_0}{2C}\hat{j}\cos(kz - wt)$

$\vec{B}_2 = \frac{-E_0}{2C}\hat{j}\cos(kz + wt)$

So, $\vec{B} = \vec{B}_1 + \vec{B}_2 = \hat{j}\frac{E_0}{2C} \times 2 \cdot \sin(kz)\sin(wt)$

25.(D) Direction of propagation + z-axis

$B_0 = \frac{E_0}{C} = 2 \times 10^{-7} T$

So only choice option (D)

26.(D) $\vec{S} = -6\hat{j} + 8\hat{k}$

$\hat{S} = \frac{\vec{S}}{|\vec{S}|} \Rightarrow \hat{S} = \frac{6\hat{j} - 8\hat{k}}{\sqrt{(6)^2 + (-8)^2}} \Rightarrow \hat{S} = \frac{6\hat{j} - 8\hat{k}}{\sqrt{100}}$

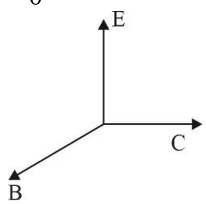
$\hat{S} = \frac{2(3\hat{j} - 4\hat{k})}{10} \Rightarrow \hat{S} = \frac{3\hat{j} - 4\hat{k}}{5}$

27.(D) $E_0 = B_0 C, C =$ speed of light in vacuum

$E_0 = 3 \times 10^{-8} \times 3 \times 10^{8} \ V/m$

$E_0 = 9 \ V/m \quad \therefore \quad \vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\hat{k} \ V/m$

28.(A)



$\hat{E} = \hat{i}$; $\quad E = BC = 15$

29.(D) B is $\perp$ to direction of propagation of e.m wave, $\quad \vec{B} = B_0\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)\cos(\omega t - kr)$

30.(B) Electric field at $\left(0, 0, \frac{\pi}{k}\right)$ is $-E_0\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

Hence force due to electric field is along $-\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

$\vec{B} = \vec{E} \times \vec{v} = \left[-\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) \times \hat{k}\right] E_0 v_0 = E_0 v_0 \left[\frac{-\hat{i} + \hat{j}}{\sqrt{2}}\right]$

Force due to magnitude filed is along $\vec{v} \times \vec{B} = \hat{k} \times \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}}\right) = -\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

Hence the net force is anti-parallel to $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

31.(B) $\overline{B}_1 = \frac{E_0}{C}\cos(\omega t - kx)\hat{k}$ & $B_2 = \frac{E_0}{C}\cos(\omega t - ky)\hat{i}$

$\overline{F}_1 = q\left[\overline{V} \times \overline{B}_1\right] + q\overline{E}_1 = q0.8E_0\hat{i} + qE_0\hat{j}$

$\overline{F}_2 = q\left[\overline{V} \times \overline{B}_2\right] + q\overline{E}_2 = -q0.8E_0\hat{k} + qE_0\hat{k} = 0.2qE_0\hat{k}$

$\overline{F} = \overline{F}_1 + \overline{F}_2 = qE_0\left[0.8\hat{i} + \hat{j} + 0.2\hat{k}\right]$

32.(C) $\hat{c} = \hat{E} \times \hat{B}$

$\hat{E} = \hat{k}$ and $\hat{B} = \frac{2\hat{i} - 2\hat{j}}{\sqrt{4 + 4}} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

Then $\hat{C} = \hat{k} \times \left(\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}} \right) = \frac{\hat{j}}{\sqrt{2}} + \frac{\hat{i}}{\sqrt{2}}$

33.(A) $\vec{B} = 3 \times 10^{-8} \sin\left[200\pi(y + ct)\right]\hat{i}T$

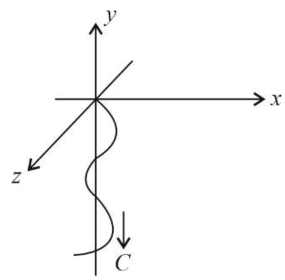
$\therefore \quad E_0 = B_0 C = 3 \times 10^{-8} \times 3 \times 10^8 = 9\ v/m$

$\because \quad \vec{B}$ is along +x-axis and propagation is along –y-axis

$\therefore \quad \vec{E}$ should be directed towards –z-axis

$\because \quad \vec{E} \times \vec{B}$ is along direction of propagation

$\therefore \quad \vec{E} = -9 \sin\left[200\pi(y + ct)\right]\hat{k}$



34.(C) $B_0 = \frac{E_0}{C} = \frac{E_0}{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}} = E_0 \sqrt{\mu_0 \varepsilon_0}$

$\vec{B} = B_0 \hat{k} \cos(\omega t - kx)$

$\vec{B}\ at\ t = 0 = B_0 \cos(-kx)\hat{k}$

$= E_0 \sqrt{\mu_0 \varepsilon_0} \cos kx\ \hat{k}$

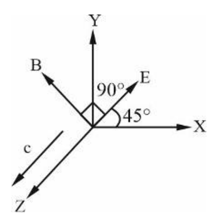
35.(B)

36.(B) $\vec{E} = b(\hat{x} + \hat{y})\sin(kz - \omega t)$

$c = \frac{E_0}{B}$

$B = \frac{E_0}{c}$

$E \times B \,||\, c$



37.(A) Because amplitude of electric field is $30\hat{j}$

Therefore amplitude of magnetic field will be $\frac{30}{c}\hat{k}$

So maximum magnetic force is

$F = qvB \sin 90$

$= 1.6 \times 10^{-19} \times 0.1c \times \frac{30}{c}$

$= 4.8 \times 10^{-19} N$

38.(C) $a \to IV$

$b \to II$

$c \to I$

$d \to III$

39.(194) $I = Cu$

when $u = \frac{1}{2}\varepsilon_0 E_0^2 = \varepsilon_0 E_{rms}^2$

$I = C\varepsilon_0 E_{rms}^2$

$\frac{315}{\pi} = 3 \times 10^8 \times \varepsilon_0 E_{rms}^2$

$\frac{4 \times 105}{4 \times \pi\varepsilon_0} \times 10^{-8} = E_{rms}^2$

$420 \times 9 \times 10 = E_{rms}^2$

$E_{rms} = 30\sqrt{42} = 194\ V/m$

40.(B) Magnitude of electric field, $E_0 = B_0 c = (1.2 \times 10^{-7})(3 \times 10^8) = 36\ V/m$

If the direction of propagation is along the unit vector $\hat{S}$, the direction of electric field is along $\hat{B} \times \hat{S}$. Therefore, in the given situation, the electric field is along –Y direction.

41.(A) $\frac{B^2}{2\mu_0} = 1.02 \times 10^{-8}$

$\because \quad \frac{\mu_0}{4\pi} = 10^{-7}$

$B^2 = 1.02 \times 10^{-8} \times 2 \times 4\pi \times 10^{-7} = 2.04 \times 4 \times 3.14 \times 10^{-15}$

$\approx 25.6 \times 10^{-15}$

25600×10^{-18}

$B = 160\ nT$